Structural Response and Acoustic Fatigue for Random Progressive Waves and Diffuse Fields

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The analysis of fatigue life under random acoustic loading is presented for honeycomb panels subjected to progressive waves and diffuse fields. The progressive wave loading simulates the environment associated with the liftoff of the Shuttle, while the diffuse loading approximates the environment in an acoustic test chamber. Numerical results are presented for honeycomb panels that allow one to readily observe the effects of various parameter changes, including the type of acoustic loading and the size and thickness of the panel. Large panels, which have low fundamental frequencies where there is little acoustic energy, respond primarily in the higher modes. For these panels, calculations indicate that for the same acoustic power spectral density, the fatigue life is longer when in the diffuse field of an acoustic test chamber than when subjected to the progressive waves of a liftoff environment.

Nomenclature

A	= panel area
a, b	= panel length and width
c	= speed of sound
D	= honeycomb panel modulus
E	= Young's modulus
$f_{mn}(x,y)$	= mode shape of the m, n mode
G_c	= honeycomb core shear modulus
G_p	= power spectral density of the acoustic load
j_{mn}	= joint acceptance
k	= acoustic wave number
k_p	= panel wave number
L_p	= sound pressure level
m, n	= mode numbers
N	= number of cycles
S	= rms stress, ksi
S_{mn}	= radiation resistance of the m, n mode
t_c	= honeycomb core thickness
t_f	= face sheet thickness
T	= fatigue life
x, y	= panel coordinates
$\bar{x}_1, \bar{x}_2$	= two points on the panel $(x_1, y_1), (x_2, y_2)$
α, β	= defined in Eq. (8)
α_ν	= defined in Eq. (16)
γ	= wave speed ratio defined in Eq. (10)
ζ	= damping ratio
θ, ϕ	= spherical coordinates
λ	= defined in Eq. (18)
μ	= panel mass per unit area
ν	= Poisson's ratio
ν_{mn}	= natural frequency of the m, n mode
ν_{eq}	= defined in Eq. (13)
ν_{peak}	= defined in Eq. (17)
ρ	= mass density
$\sigma_{q_{mn}}$	= stress per unit deflection in m, n mode
σ_{mn}	= rms stress in m, n mode

Introduction

THIS paper presents an analysis of the effect of random progressive acoustic waves vs diffuse fields on honeycomb panels. Of particular interest is how an acoustic fatigue test specimen performs in a reverberant acoustic test chamber, as opposed to how it performs in a progressive wave environment. Very large test chambers tend to have reverberant acoustic fields that are nearly diffuse. The assumption of a diffuse field will be used in this study to approximate the environment of a large reverberant test chamber.

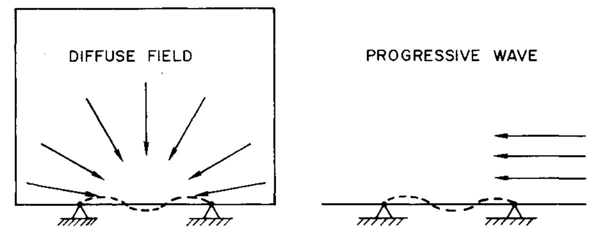

Fig. 1 Acoustic fatigue life tests for a diffuse field and a progressive wave field.

In order to study the effect of these two types of acoustic environments on the fatigue life of a structure, a reference structure is needed. For simplicity, a plane honeycomb panel is chosen. The panel is simply supported in a baffle as shown in Fig. 1. The size and thickness of the panel is varied so that the trends of fatigue life in the face sheet at the center of the panel can be seen for both diffuse and progressive acoustic wave excitations.

Fundamental Equations of the Acoustic Fatigue Model

From the work of Powell,[1,2] the fundamental equation for the rms stress response of a structural vibration mode is

$$\sigma_{mn}^2 = \frac{\nu_{mn} G_p(\nu_{mn}) \sigma_{q_{mn}}^2(x,y) A^2 j_{mn}^2}{[4/\pi]\zeta_{mn}[2\pi\nu_{mn}]^4 \left[\int_A \mu(x,y) f_{mn}^2(x,y)\, dA\right]^2} \quad (1)$$

where j_{mn}, the joint acceptance between the acoustic field and the mode shape, is defined by

$$j_{mn}^2 = \frac{1}{A^2}\int_A\int_A f_{mn}(\bar{x}_1) f_{mn}(\bar{x}_2)\rho(\bar{x}_1, \bar{x}_2, \omega)\, dA_1\, dA_2 \quad (2)$$

and $\rho(\bar{x}_1, \bar{x}_2, \omega)$ is the spectrum of the correlation coefficient.

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Honeycomb Panel

The panel, as shown in Fig. 2, is assumed to be simply supported in an infinite baffle. The mode shapes for the panel are of the form

$$f_{mn} = \sin(m\pi x/a)\sin(n\pi y/b) \tag{3}$$

where $0 \le x \le a$ and $0 \le y \le b$.

The natural frequencies of the honeycomb panel, corresponding to the above mode shapes are[3]

$$\nu_{mn} = \frac{\pi}{2}\sqrt{\frac{D/\mu}{1 + \frac{D\pi^2}{t_c G_c}\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)}}\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right) \tag{4}$$

where

$$D = \frac{t_f(t_c + t_f)^2 E}{2(1 - \nu^2)}$$

$$\mu = \rho_{\mathrm{Al}} 2t_f + \rho_c t_c$$

The x component of normal stress in the face sheet per unit deflection in the mode is given by

$$\sigma_{q_{mn}} = \frac{\frac{D\pi^2}{t_f(t_c + t_f)}\left(\frac{m^2}{a^2} + \nu\frac{n^2}{b^2}\right)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}}{1 + \frac{D\pi^2}{t_c G_c}\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)} \tag{5}$$

where $b \ge a$.

Acoustic Field

The power spectral density of the acoustic field is similar to that prescribed for several of the acoustic tests on shuttle structure and is shown in Fig. 3.[4] The equations fitting these prescribed data are as follows.

$$\nu_{mn} G_p = 3.565 \times 10^{-17} 10^{L_p/10} \text{ psi}^2 \tag{6}$$

where

$$L_p = -20.4(\log_{10}\nu_{mn})^2 + 85.7\log_{10}\nu_{mn} + 65.1 \text{ for } \nu_{mn} \le 250 \text{ Hz}$$

and

$$L_p = 175.8 - 9.385\log_{10}\nu_{mn} \text{ for } \nu_{mn} > 250 \text{ Hz}$$

For the case of a plane progressive acoustic wave, the spectrum of the correlation coefficient is

$$\rho(x_1', x_2', \omega) = \cos[k(x_2' - x_1')\sin\theta] \tag{7}$$

where x' is a coordinate on the panel in the direction of the acoustic wave propagation and θ the angle of incidence of the acoustic wave.

If we take ϕ as the angle between x' and x, then

$$x_2' - x_1' = (x_2 - x_1)\cos\phi + (y_2 - y_1)\sin\phi$$

and

$$\rho = \cos[k\{(x_2 - x_1)\cos\phi + (y_2 - y_1)\sin\phi\}\sin\theta] \tag{7a}$$

where (x_1, y_1) and (x_2, y_2) are two points on the panel surface.

Joint Acceptance

The joint acceptance between the panel vibration modes and the plane progressive acoustic wave is found from Eqs. (2), (3) and (7a). Thus

$$j_{mn}^2 = \frac{16}{\pi^4}\left(\frac{\sin(\alpha/2 + m\pi/2)}{m[(\alpha/m\pi)^2 - 1]}\right)^2\left(\frac{\sin(\beta/2 + n\pi/2)}{n[(\beta/n\pi)^2 - 1]}\right)^2 \tag{8}$$

where

$$\alpha = ka\sin\theta\cos\phi$$

$$\beta = kb\sin\theta\sin\phi$$

$$k = \omega/c = 2\pi\nu_{mn}/c$$

For the case of the diffuse sound field, the joint acceptance is averaged over all spatial directions in the half-space over the panel. Thus

$$\langle j_{mn}^2 \rangle_\Omega = \frac{2}{\pi}\int_0^{\pi/2}\int_0^{\pi/2} j_{mn}^2(\phi, \theta)\sin\theta \, d\theta \, d\phi \tag{9}$$

This problem has been considered by Smith and Lyon.[5] Through the principle of reciprocity, they have shown that the spatial average of the joint acceptance is related to the radiation resistance $S_{mn}(\gamma)$ of the structure. Upon substituting for j_{mn}^2 from Eq. (8), it can be observed that the integral in Eq. (9) is the same as that evaluated in previous work on the radiation resistance of simply supported panels.[6] This work provides curves of the radiation resistance that could be used with the relationship

$$\langle j_{mn}^2 \rangle_\Omega = \frac{c^2 S_{mn}(\gamma)}{8\pi ab\nu_{mn}^2} \tag{10}$$

where the wave number ratio is

$$\gamma = k/k_p$$

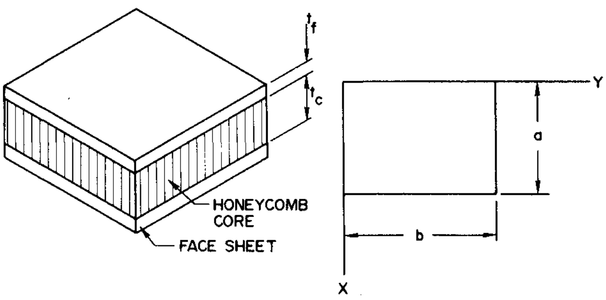

Fig. 2 Honeycomb panel.

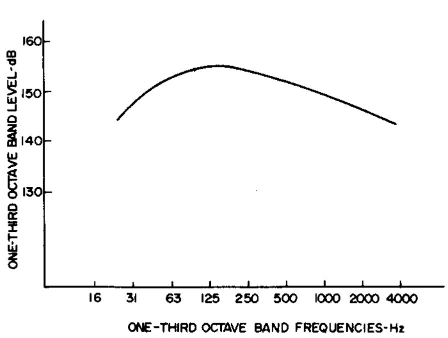

Fig. 3 Sound pressure level L_p of the acoustic excitation.

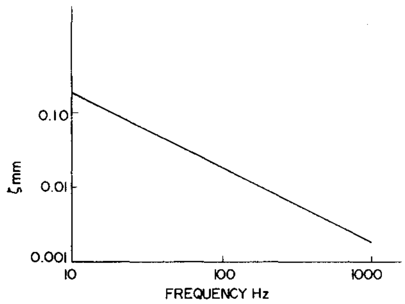

Fig. 4 Damping ratio of the panel modes as a function of the modal frequencies.

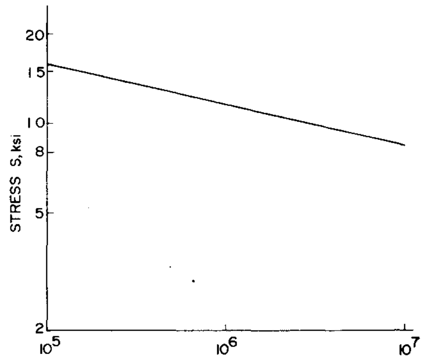

Fig. 5 Narrow-band ($\alpha_\nu = 1$) random fatigue stress cycle relationship for 2024-T851 aluminum.

and

$$k_p = \pi\sqrt{(m/a)^2 + (n/b)^2}$$

Maidanik,[7] Leffington et al.,[8] Williams,[9] and the author[6] have presented asymptotic solutions for the radiation resistance that could be used in the appropriate wave number ratio range. However, the most direct approach is to integrate Eq. (9) numerically.

Structural Damping

The structural damping shown in Fig. 4 is taken as a decreasing function of frequency as given by Hay.[10] The equation for the damping is

$$\zeta_{mn} = D_0 \nu_{mn}^{-D_I} \tag{11}$$

where $D_0 = 1.90$ and $D_I = 1.00$.

Response Computation

From the above description of the panel and the acoustic excitation, one can compute the mean square stress for each mode responding at its natural frequency ν_{mn}. The overall mean square stress response is obtained by summing the contribution from each mode,

$$\sigma_{\rm rms}^2 = \sum \sigma_{mn}^2 \tag{12}$$

The equivalent frequency of this overall response is given by[11,12]

$$\nu_{\rm eq} = \sqrt{\frac{\sum \sigma_{mn}^2 \nu_{mn}^2}{\sum \sigma_{mn}^2}} \tag{13}$$

Fatigue Analysis

The narrow-band random fatigue stress cycle relationship for 2024-T851 aluminum is given by

$$NS^{7.2} = 4.335 \cdot 10^{13} \tag{14}$$

where the stress S is in ksi. This relationship is shown in Fig. 5.

Thus, the fatigue life using the narrow-band fatigue relationship is

$$T = N/\nu_{\rm eq} \tag{15}$$

In order to take the wide-band character of the stress power spectral density (PSD) into account, the work of Wirsching[11] is utilized. Wirsching studied the effect of the shape of the stress PSD on fatigue life by simulating the stress-time history and performing a rainflow count. He presented a correction factor based on the irregularity factor

$$\alpha_\nu = \nu_{\rm eq}/\nu_{\rm peak} \tag{16}$$

where $\nu_{\rm peak}$ is the stress peak frequency given by[11,12]

$$\nu_{\rm peak} = \sqrt{\frac{\Sigma \sigma_{mn}^2 \nu_{mn}^4}{\Sigma \sigma_{mn}^2 \nu_{mn}^2}} \tag{17}$$

For narrow-band response, i.e., primarily in only one vibration mode, $\nu_{\rm peak} = \nu_{\rm eq} = \nu_{mn}$ and $\alpha_\nu = 1$. For multimode response, $\nu_{\rm peak}$ is greater than $\nu_{\rm eq}$ and $\alpha < 1$. For 2024-T851 aluminum, the equation associated with Wirsching's correction is

$$\lambda = [0.688] + [3.12]\left[1 - \sqrt{1-\alpha^2}\right]^{9.10} \tag{18}$$

where the corrected fatigue life is

$$T(\alpha_\nu) = T(\alpha_\nu = 1)/\lambda \tag{19}$$

Panel Property Values

The property values assumed in the study are

$\rho_{\rm Al} = 0.10$ lbm/in.^{3}

$E = 10.3 \times 10^6$ psi

$\nu = 0.32$

$\rho_c = 0.0025$ lbm/in.^{3} for honeycomb panel core

$t_f = 0.024$ in. for the honeycomb panel face sheet thickness

$t_c = 0.60$ in. for the honeycomb panel core thickness

$G_c = 60 \times 10^3$ psi for honeycomb panel core sheet modulus

Computational Results

A square panel, 80 in. on a side, is analyzed for both the progressive wave environment and the diffuse field environment. The direction of the progressive wave is assumed to be grazing ($\theta = 90$ deg) and in the x coordinate direction ($\phi = 0$). The stress and fatigue life are calculated at the center of the panel where $x/a = 0.5$ and $y/b = 0.5$.

From Eq. (12), the rms stress is determined. The cumulative distribution of this stress is shown in Fig. 6 and demonstrates that the progressive wave field is more effective in generating response in the higher frequency modes than is the diffuse field.

The equivalent frequency, or the frequency of positive zero crossing, is shown in a cumulative distribution in Fig. 7. Here

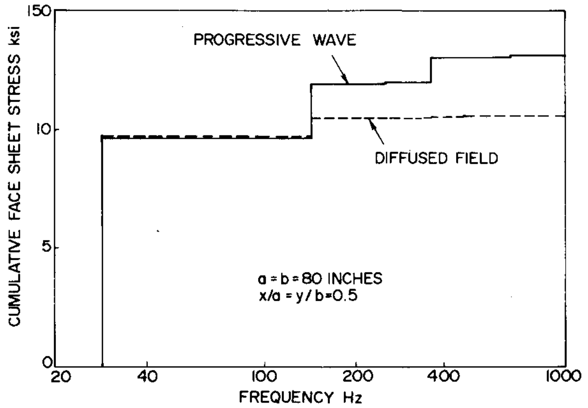

Fig. 6 Cumulative face sheet rms stress at panel center for the square 80 in. panel with both progressive wave and diffuse field excitation.

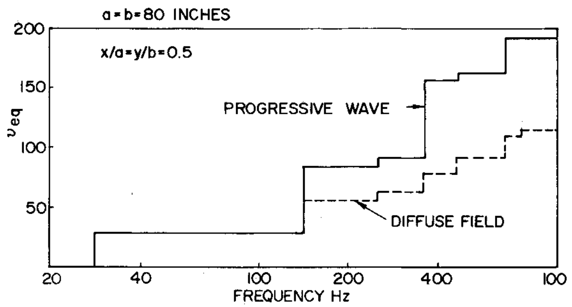

Fig. 7 Cumulative equivalent frequency ν_{eq} of the face sheet at the center of a square 80 in. panel.

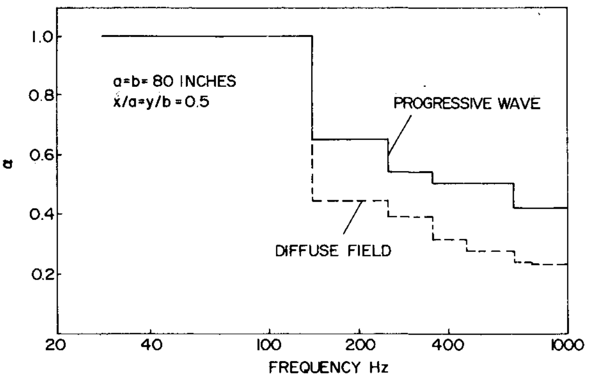

Fig. 8 Cumulative irregularity function α_ν for the face sheet at the center of a square 80 in. panel.

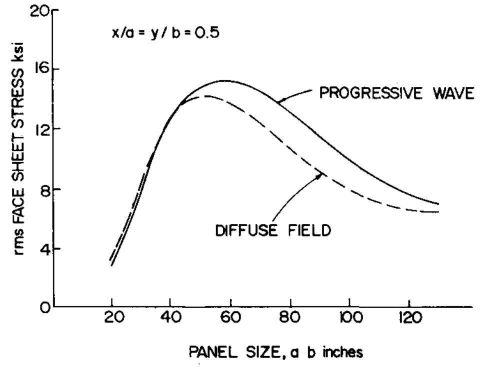

Fig. 9 Rms face sheet stress as a function of panel size for both progressive wave and diffuse field excitation.

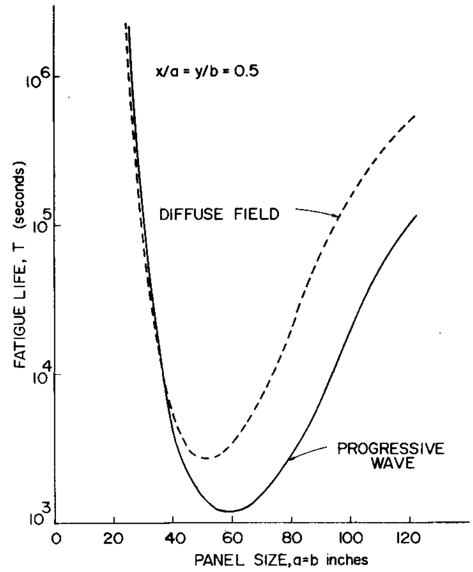

Fig. 10 Acoustic fatigue life of the face sheet at the center of the panel as a function of panel size.

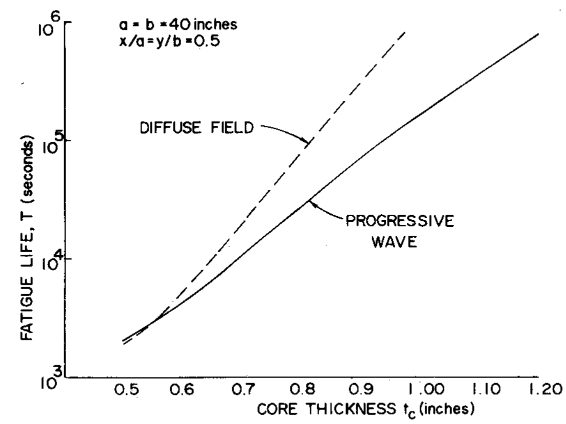

Fig. 11 Acoustic fatigue life of the face sheet at the center of a square 40 in. honeycomb panel.

we see that the stronger response of the higher-frequency modes under progressive wave excitation increases the equivalent frequency more than does the diffuse field environment, thus contributing further to a decrease in fatigue life.

The cumulative distribution of the irregularity factor α_ν is shown in Fig. 8. Applying Wirsching's correction for wide-band response from Eqs. (13) and (14), we find that $\lambda = 0.688$ for both cases and thus does not affect the comparison of fatigue life for the two kinds of acoustic excitation.

Considering the dimension of a square panel as a variable, the effect of panel size on face sheet stress and fatigue life can be studied. These results are presented in Figs. 9 and 10. For the case of an 80 in. panel, the ratio of panel life for diffuse vs progressive wave excitation is 6.95. The maximum value of this ratio (8.61) occurs at a panel size of 97 in.

Holding the panel size constant, the effect of varying the core thickness on fatigue life can be observed. In Fig. 11, the panel size is $a = b = 40$ in., while in Fig. 12 the panel size is $a = b = 80$ in. For the 40 in. panel, the life is about the same for a thin core of 0.50 in.; but as the core thickness increases, the life under diffuse loading increases faster than does the life under progressive wave loading. For the 80 in. panel, the life under diffuse loading is always greater than under progressive wave loading.

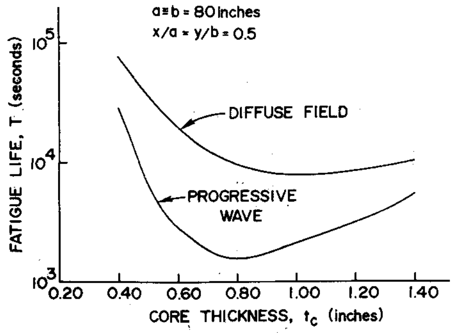

Fig. 12 Acoustic fatigue life of the face sheet at the center of a square 80 in. honeycomb panel.

Discussion of Results

The small, thin panel ($a = b < 40$ in. and $t_c < 0.60$) responds primarily in its fundamental mode and shows little sensitivity to whether the acoustic field is diffuse or part of a progressive wave. For larger panels, the fundamental mode has a low frequency where there is little acoustic energy available to drive it. In addition, the assumed damping is larger at these low frequencies. Thus, much of the fatigue damage is caused by the response of the higher-frequency modes. For the case of thicker panels, the fundamental mode responds more strongly, but the higher-frequency modes also contribute significantly. The higher-frequency modes that strongly couple with the sound field have bending wave speeds which are near the speed of sound in air. For these modes, the correlation of the progressive waves determines a higher joint acceptance than does that of the diffuse field. Thus, a progressive wave environment is more damaging to panels with significant response above the fundamental frequency than is the diffuse field.

Conclusions

This study is intended to provide insight into what changes in dynamic response and fatique life occur when the acoustic field driving the structure is changed from that found in a large reverberant chamber to the kind of field associated with the liftoff environment. The analysis demonstrates the fatigue life in a reverberant field is longer than the life in a progressive wave field of identical power spectral density. Thus, one must conclude that the reverberant chamber test is likely to be nonconservative and quite misleading. For structure that must perform in a progressive wave environment, it is recommended that a progressive wave field be used in the acoustic test.

Acknowledgments

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